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(54) **Coding for the reduction of peak to average power ratio in multicarrier modulation systems**

(57)   The encoding/transmission of information in an OFDM system is enhanced by using complementary codes. The complementary codes, more particularly, are converted into phase vectors and the resulting phase vectors are then used to modulate respective carrier signals. The modulated result is then transmitted to a receiver which decodes the received signals to recover the encoded information.

FIG. 1
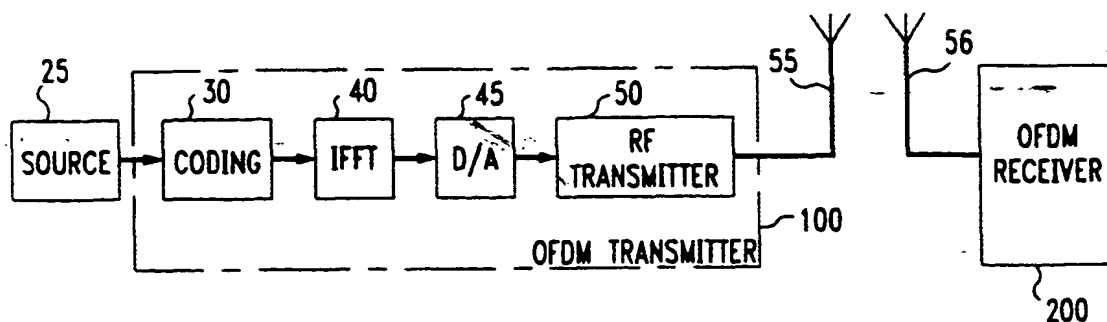
Description

FIELD OF THE INVENTION

The invention relates to the modulation of data for transmission in a digital communications system.

BACKGROUND OF THE INVENTION

In a system employing Orthogonal Frequency Division Multiplexing (OFDM), groups of kN bits are typically transmitted simultaneously over N subchannels, with k bits per channel using some form of Quadrature-Amplitude Modulation. If N is made large enough at a constant bit rate, then a subchannel may experience minimum intersymbol interference, but may still be subject to narrowband fading. The effect of such fading may be different for each subchannel. Also, when N sinusoidal signals respectively defining the subchannels are summed with the same phase for transmission in an OFDM system, the result creates a peak-to-average power (PAP) ratio that is typically N times larger than the average power level used in the transmission of a single symbol. Because of this; an OFDM transmitter has to use a linear power amplifier having a large "backoff" corresponding to the PAP ratio. Disadvantageously, then, a decrease in efficiency occurs as the PAP ratio increases. This problem is especially acute when OFDM is used in portable devices where power efficiency is a key concern.

SUMMARY OF THE INVENTION

We have recognized that the aforementioned problem may be dealt with using so-called complimentary codes that have been modified in accord with particular phase modulation. Specifically, M input phases, e.g., four phases, directly related to input data may be encoded into N, e.g., eight, output phases associated with respective carrier signals, in which a phase shift, θ_i, is applied to a carrier (subchannel) to achieve a low PAP ratio, e.g., 3 dB.

A receiver receiving the transmitted signal demodulates the N carriers and, by using a fast Fourier transform, obtains N vectors respectively defining the N phases and thus the input data. In the event that a number of carrier signals, e.g., three, are lost during the transmission, the input data may still be recovered in accord with an aspect of the invention.

These and other aspects of the invention will be appreciated from the ensuing detailed description when read in conjunction with drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

In the drawings:

FIG. 1 illustrates in block diagram form a wireless system composed of a transmitter and receiver arranged in accord with the principles of the invention; and

FIG. 2 illustrates in more detail the receiver of FIG. 1.

DETAILED DESCRIPTION

In accordance with an illustrative embodiment of the invention, a set or sequence of complimentary codes of a desired length, e.g., a length 8 code, may be generated starting with a so-called kernel of the desired length. One possible kernel for generating codes of length 8 may be, for example, the series {111-111-11}. (The rules for generating a kernel are discussed in the article entitled "Complementary Series", by M. J. E. Golay, published in the IRE Transactions on Information Theory, vol. IT-7, pp. 82-87, April, 1961, which is hereby incorporated by reference.) Once a particular kernel has been selected, then independent phase rotations are applied to the elements (bits) forming the selected kernel. This may be done using a particular transformation comprising, e.g., eight columns, each column having a different group of individual ones of a predetermined number of phases, e.g., four phases, ϕ_1, ϕ_2, ϕ_3 and ϕ_4, as follows:

$$\begin{array}{cccccccc}
\varphi_1 & \varphi_1 & \varphi_1 & \varphi_1 & \varphi_1 & \varphi_1 & \varphi_1 & \varphi_1 \\
\varphi_2 & 0 & \varphi_2 & 0 & \varphi_2 & 0 & \varphi_2 & 0 \\
\varphi_3 & \varphi_3 & 0 & 0 & \varphi_3 & \varphi_3 & 0 & 0 \\
\varphi_4 & \varphi_4 & \varphi_4 & \varphi_4 & 0 & 0 & 0 & 0
\end{array}$$

The phase angles (also referred to herein as constellation symbols or just symbols), φ_i, forming each of the columns are then applied to the elements of the kernel to form a complementary code, as shown by the following expression:

$$c = \{e^{j(\varphi_1+\varphi_2+\varphi_3+\varphi_4)}, e^{j(\varphi_1+\varphi_3+\varphi_4)}, e^{j(\varphi_1+\varphi_2+\varphi_4)}, -e^{j(\varphi_1+\varphi_4)}, e^{j(\varphi_1+\varphi_2+\varphi_3)}, e^{j(\varphi_1+\varphi_3)}, -e^{j(\varphi_1+\varphi_2)}, e^{j\varphi_1}\} \quad (1)$$

The vectors forming the complementary code may be represented by respective output phases θ_1 through θ_8 and may be formed as illustrated by the following transformation derived in accordance with equation 1:

$$\theta_1 = \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4$$

$$\theta_2 = \varphi_1 + \varphi_3 + \varphi_4$$

$$\theta_3 = \varphi_1 + \varphi_2 + \varphi_4$$

$$\theta_4 = \varphi_1 + \varphi_4 + \pi$$

$$\theta_5 = \varphi_1 + \varphi_2 + \varphi_3$$

$$\theta_6 = \varphi_1 + \varphi_3$$

$$\theta_7 = \varphi_1 + \varphi_2 + \pi$$

$$\theta_8 = \varphi_1$$

The eight phases θ_1 through θ_8 may then be used to respectively modulate eight OFDM-subcarriers as is done conventionally in an OFDM system, as discussed below.

Specifically, assume that OFDM transmitter 100, FIG. 1, embodying the principles of our invention receives via encoder circuit 30 a stream of data bits from a source of data bits 25. Encoder circuit 30, which may be, for example, a conventional digital signal processor, partitions the data stream as it is received into successive groups of twelve bits each group and stores each such group as it is formed in internal memory (not shown). Encoder circuit 30 then unloads a group of stored data bits from the internal memory, encodes the data bits in accordance with the principles of the invention and supplies the result to Inverse Fast-Fourier Transform (IFFT) processor 40. Encoder circuit 30, more particularly, first encodes the group of twelve bits that it unloads from internal memory into, for example, four 8-PSK (phase shift keying) phases as is done conventionally for 8-PSK in general. For example, subgroups of three bits (also referred to herein as a data word) may be converted to 8-PSK using so-called Gray scale encoding such that

the subgroup 0,0,0 is encoded as 0; 0,0,1 is encoded as π/4; 0,1,1 as π/2, and so on as illustrated by the following translation table.

bits	phase
000	0
001	π/4
011	π/2
010	3π/4
110	π
111	5π/4
101	6π/4
100	7π/4

Encoder circuit 30 associates the four subgroups of data bits of a group with a respective one of the constellation symbols φ_i. That is, encoder 30 associates the first subgroup of three bits of a group of bits with the symbol φ_1, and associates the next (second) subgroup of three bits with the symbol φ_2, and so on. For example, assume that a group is composed of the following series of bits: 11101010001. The subgroup encoding and φ_i symbol associations based on the above translation table stored in memory would be as follows:

111	5π/4	φ_1
010	3π/4	φ_2
100	7π/4	φ_3
001	π/4	φ_4

Encoder circuit 30 then generates θ_1 through θ_8 in accord with the above transformations for θ_i. For example, as indicated above $\theta_1 = \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4$, then, for the present illustrative example, $\theta_1 = 5\pi/4 + 3\pi/4 + 7\pi/4 + \pi/4$. Similarly, $\theta_2 = 5\pi/4 + 7\pi/4 + \pi/4$; $\theta_3 = 5\pi/4 + 3\pi/4 + \pi/4$; and so on. Note that the values of symbols φ_1 through φ_4 would be different for a subgroup having a different combination of twelve bits, e.g., 000101110011. However, note that symbol φ_1, regardless of its value, is associated with the first subgroup of bits of a group, and symbol φ_4 is associated with the last subgroup of bits of that group. When encoder circuit 30 has completed the generation of the eight phases θ_1 through θ_8, it then supplies the values for those phases to IFFT processor 40, which may be, for example, a conventional digital signal processor (DSP). Moreover, the DSP that implements the encoder 40 function may be programmed to also implement the IFFT 40 function. IFFT processor 40, more particularly, converts the data from the time domain to the frequency domain using the inverse of the Fast Fourier transform to generate respective phase vectors. Processor 40 then modulates a plurality, e.g., eight, digital carriers respectively using the values of the eight phase vectors formed by θ_1 through θ_8. That is, IFFT processor 40 modulates a carrier$_i$ (channel$_i$) using the value of a respective phase vector θ_i. IFFT processor 40 then outputs the result to conventional analog to digital converter circuit 45, which converts the digital signals it receives from IFFT processor 40 to analog signals. The analog signals are then supplied to RF transmitter 50 which modulates the analog signals onto an RF carrier, e.g., a 5.2 GH$_3$ carrier, and supplies the result to antenna 55 for transmission to wireless type receivers, e.g., receiver 200. Encoder circuit 30 (OFDM Transmitter 100) then goes on to similarly process the next the group of data bits stored in the aforementioned internal memory.

Receiver 200, FIG 2, more particularly, includes a conventional RF section 230 for receiving the resulting composite signal via antenna 256 and processing (downconverting) the signal in a conventional manner as it is received. The processed result is then supplied to conventional analog to digital converter 245 which converts the processed result to corresponding digital signals. The digital signals are then supplied to Fast Fourier Transform (FFT) processor 240 which demodulates the N carriers. Processor 240 does this by performing a FFT on the digital signals supplied by converter 245. The output of FFT processor 240 comprises N (where N = eight for the present illustrative example) vectors (in-phase and quadrature samples), representing the amplitudes and phases of the N different subchannels, as illustrated by the following expression:

$$r_i = e^{(5\pi/4 + 3\pi/4 + 7\pi/4 + \pi/4)}, e^{j(5\pi/4 + 7\pi/4 + \pi/4)}, e^{j(5\pi/4 + 3\pi/4 + \pi/4)},$$

$$-e^{j(5\pi/4 + \pi/4)}, e^{j(5\pi/4 + 3\pi/4 + 7\pi/4)}, e^{j(5\pi/4 + 7\pi/4)}, -e^{j(5\pi/4 + 3\pi/4)}, e^{j5\pi/4} \qquad (2)$$

The N vectors are supplied to decoder 230 which then decodes the output of the FFT to determine the values of the respective phase symbols φ_i. Such decoding, in accord with an aspect of the invention, is applied to alternating elements of the complementary code in which each of the complex odd samples of the FFT output is multiplied against a paired complex conjugate of the even samples. A summation of the result of each multiplication forms a vector which has the value of the sought-after phase angle (symbol φ_i). This procedure may be followed for even and odd pairs of the samples as well as quads, etc. Decoder 230, more particularly and in accordance with this decoding technique, combines r_i with a complex conjugate of the kernel code used to form the complementary code in the encoding of the group of data bits at the transmitter. One such kernel code may be, for example, {111-111-11}, as mentioned above. If that is the case, then r_4 and r_7 are inverted -- meaning that the sign of the resulting multiplication for r_4 and r_7 is positive. (For notational purposes, the following equates the received digital signals (samples) with x_i, respectively.) Decoder 230 then generates three vectors y_2, y_3 and y_4 as a function of respective ones of the digital samples x_i.

More specifically, and as discussed above, transmitter 100 encodes phases φ_1 to φ_4 into eight phases θ_1 through θ_8 in accord with the aforementioned transformation. In addition, receiver 200 removes the effect of the kernel code on the received signal by multiplying the elements of vector r_i by the kernel code and expressing the phase encoding in matrix form as follows:

$$\theta = A\varphi$$

where θ and φ are vectors containing the values of the eight phases θ_i and four φ_i phases and A is an encoding matrix as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Using the matrix, receiver 200 may then determine the values of the encoded phases φ_i from the measured phases θ_i by determining a least-squares solution for the linear equations represented by the matrix as follows:

$$\varphi = P\theta, \quad P = (A^T A)^{-1} A^T = \frac{1}{4} \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 1 & 1 & 2 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

where P is the pseudo-inverse of A and superscript T denotes a matrix transpose. Unfortunately, such equations are not completely linear, since the phase values are modulo 2π. Because of this, the above method cannot be applied directly to determining the values for φ_i. However, an inspection of the A-matrix reveals that the phases φ_2, φ_3 and φ_4 may be determined as a summation of 4 subtracted θ_i pairs, as mentioned above. For example, each of the subtractions θ_1-θ_2, θ_3-θ_4, θ_5-θ_6 and θ_7-θ_8 provide a value for φ_2. A preferable way to get the phase difference between two vectors is to multiply one vector with the complex conjugate of the other vector. Doing so leads to the inventive decoding procedure in which three vectors y_2, y_3, y_4 are determined as follows:

$$y_2 = x_1 x^*_2 + x_3 x^*_4 + x_5 x^*_6 + x_7 x^*_8$$

$$y_3 = x_1 x_3^* + x_2 x_4^* + x_5 x_7^* + x_6 x_8^*$$

$$y_4 = x_1 x_5^* + x_2 x_6^* + x_3 x_7^* + x_4 x_8^*$$

where * means complex conjugate and where the arctan of the angle between the real and imaginary parts of each term respectively forming vectors y_2 through y_4 provides the value of the corresponding phase symbol φ_2 through φ_4, respectively. (It is noted that the decoding technique used to derive the pairing of the elements forming each term of each of the above vectors may also be determined by inspection in which the difference between each pair of elements provides the value of the sought after phase angle.) For example, the value of vector y_2 is determined as follows for the instant illustrative example of the invention:

$$y_2 = (e^{(5\pi/4 + 3\pi/4 + 7\pi/4 + \pi/4)} e^{-j(5\pi/4 + 7\pi/4 + \pi/4)}) + (e^{j(5\pi/4 + 3\pi/4 + \pi/4)}$$

$$e^{-j(5\pi/4 + \pi/4)}) + (e^{j(5\pi/4 + 3\pi/4 + 7\pi/4)} e^{-j(5\pi/4 + 7\pi/4)}) + (e^{j(5\pi/4 + 3\pi/4)} e^{-j5\pi/4})$$

$$= e^{3\pi/4} + e^{3\pi/4} + e^{3\pi/4} + e^{3\pi/4}$$

A determination of the value of each term of vector y_2 thus leads to a determination of the phase value of φ_2, which, for the present illustrative example is $3\pi/4$. In practice, the determination would be an estimate of φ_2. Decoder 230 deal with that problem by "rounding off" the estimated value to the nearest constellation phase selected for the encoding at transmitter 30, e.g., 8-PSK constellation, to generate a more accurate value for φ_2.

Decoder 230 then similarly generates phase estimates of φ_3 and φ_4 as a function of vectors y_3 and y_4, respectively, and "rounds off" those estimates in a similar manner. Doing so, yields, in accord with the present illustrative example, phase values of $7\pi/4$ and $\pi/4$ for φ_3 and φ_4, respectively.

Once decoder 230 has determined the phase values of φ_2, φ_3 and φ_4 it may then determine the value of φ_1. However note that the phase φ_1 is present in all θ_i equations as illustrated by the above transformation table. Conse- quently, φ_1 cannot be expressed as a subtraction of two θ_i values, as was done for the other phases. However, since all phases except for φ_1 can be determined in the manner discussed above, then the values for those phases can simply be substituted in the θ_i equations to create eight equations with only one unknown as one way of obtaining eight estimates for φ_1. Receiver 200 may then take the average value of the eight estimates for φ_1 to improve the Signal- to-Noise Ratio (SNR) for φ_1. It is noted that, in practice, the average may be based on only four of the estimated values, since the noise in the other four solutions is correlated with the noise in the solutions that are used.

Thus, a vector y_1 corresponding to the sought-after phase may be obtained by substituting the estimated values of the selected four solutions (phases) as follows:

$$y_1 = x_4 e^{-j\varphi_4} + x_6 e^{-j\varphi_3} + x_7 e^{-j\varphi_2} + x_8$$

$$= e^{j(5\pi/4 + \pi/4)} e^{-j\varphi_4} + e^{j(5\pi/4 + 7\pi/4)} e^{-j\varphi_3} + e^{j(5\pi/4 + 3\pi/4)} e^{-j\varphi_2} + e^{j5\pi/4}$$

$$= e^{j5\pi/4} + e^{j5\pi/4} + e^{j5\pi/4} + e^{j5\pi/4}$$

As was the case for φ_2, the arctan of the real and imaginary part of each term of vector y_1 leads to a determination of the corresponding phase value of φ_1, which, for the present illustrative example would turn out to be $5\pi/4$. Similarly, in practice, the determination would be an estimate of φ_1 as was the case for φ_1, φ_3 and φ_4. Accordingly, then, decoder 230 determines in a conventional manner the actual value of φ_1 as a function of its estimated value, i.e., decoder 230 "rounds off" the estimated value to the nearest constellation phase selected for the encoding at transmitter 30, e.g., an 8-PSK constellation.

As a result of foregoing process, receiver decoder 230 determines the data values respectively represented by symbols φ_1 through φ_4. Namely, the series of bits 11101010001 assumed above for the present example illustrating of the principles of the invention.

As mentioned above, the pairing of the elements forming each term of each of the above vectors y_i may be determined by inspection in which the difference between each such pair of elements provides the sought after phase angle. It is seen from the above, that each vector y_i comprises a plurality of such terms. Advantageously, then, receiver 200 may still determine a sought-after phase even though one or more terms forming the associated vector y_i were lost for whatever reason, e.g., due to a momentary change in the transmission environment. For example, assume that the first three channels are lost such that receiver 200 obtains only eight samples of the information transmitted by transmitter 100 as noted by the following:

$$r_i = 0, 0, 0, -e^{i(\varphi_1+\varphi_4)}, e^{i(\varphi_1+\varphi_2+\varphi_3)}, e^{i(\varphi_1+\varphi_3)}, -e^{i(\varphi_1+\varphi_2)}, e^{i\varphi_1}$$

As mentioned above, decoder 230 multiplies r_i by the kernel code used in the decoding of the transmitted data at transmitter 100. Even though the first three channels were lost -- meaning that the values of samples x_1 through x_3 would be zero -- receiver 200, nevertheless, may still recover the values of phase symbols φ_1 through φ_4 from the samples that it is able to generate as illustrated by the following:

$$y_2 = x_5 x^*_6 + x_7 x^*_8 = \varphi_2$$

$$y_3 = x_5 x^*_7 + x_6 x^*_8 = \varphi_3$$

$$y_4 = x_4 x^*_8 = \varphi_4$$

$$y_1 = x_4 e^{-i\varphi_4} + x_6 e^{-i\varphi_3} + x_7 e^{-i\varphi_2} + x_8$$

Advantageously, then, decoder 230, in accord with the principles of the invention, may still recover data that transmitter 100 transmits via a plurality of channels even though the content of one more of the channels is lost prior to being received by receiver 200.

The foregoing is merely illustrative of the principles of the invention. Those skilled in the art will be able to devise numerous arrangements, which, although not explicitly shown or described herein, nevertheless embody those principles that are within the spirit and scope of the invention. For example, although an illustrative embodiment of the invention was discussed in the context of a code of length 8, multiples of that number may be used in systems employing more than eight subchannels, e.g., sixteen subchannels. In such a system, several codes of length 8 may be interleaved to modulate the information transmitted over the channels. Such interleaving may be achieved by using one code for odd numbered channels and another code for even numbered channels. As another example, for a code length of 2^n, there will be n+1 encoded phases φ_i, which may be applied to the entire code or to alternating elements, quads, etc. Thus, the coding and decoding would be similar to the length 8 code, except for having a different number of phases φ_i. As a further example, the use of complementary codes in accordance with the principles of the invention is also applicable to "forward error correction" as well as PAP reduction coding in OFDM systems. It is also possible to do fallback rates (decreased data rates with larger coverage) by increasing the code length (using length 16 or 32 codes instead of a length 8 code) or by decreasing the number of phases (e.g., using BPSK instead of 8-PSK).

Claims

1. A method of encoding data for transmission to a receiver comprising the steps of

selecting a kernel code formed from a predetermined number of bits and applying independent phase rotations, φ_i, to each of said bits as a function of a predetermined transformation to generate respective complementary codes, θ_i,

associating a predetermined number of groups of stored bits with respective ones of said phase rotations, and

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converting the complementary codes into complex vectors, and

modulating carrier signals using respective ones of said vectors and transmitting the result to a receiver.

2. The method of claim 1 wherein said transformation comprises a number of rows corresponding to the number of said independent phase rotations and a number of columns corresponding to said predetermined number, and wherein said step of applying includes the step of applying the phase rotations forming said columns to respective ones of the said bits to form said vectors.

3. The method of claim 1 wherein said step of associating includes the steps of encoding each of said groups of bits as respective phase angles in accordance with gray scale encoding and associating the encoded phase angles with predetermined ones of said phase rotations forming said complementary codes.

4. The method of claim 1 further comprising at said receiver the steps of

receiving a composite signal of the modulated signals transmitted by said transmitter and regenerating individual ones of said vectors as a function of the received composite signal,

applying said kernel code to said regenerated vectors,

generating a number of vectors, y_i, from pairs of elements forming the regenerated vectors, in which one element in each of said pair is taken as a complex conjugate so that said one element may be subtracted from the other element of the respective pair of elements, said pairing of said elements being performed in accordance with the contents of a predetermined matrix,

deriving a respective one of said phase rotations as a function for each subtracted result forming a respective one of said vectors, y_i, said vectors, y_i, being associated with respective ones of said phase rotations, φ_i, and

deriving each phase rotation, φ_n, not associated with a respective vector as a function of the derived phase rotations and a complementary code, θ_k, formed in part by that phase rotation, φ_n.

5. The method of claim 4 further comprising the steps of identifying the groups of bits respectively associated with the derived phase rotations.

6. A method of encoding data for transmission to a receiver, said method comprising the steps of

encoding data words that are to be transmitted to said receiver into respective constellation symbols,

generating a plurality of complementary codes as a function of a selected kernel code and a predetermined transformation matrix of said constellation symbols, and

modulating a plurality of carrier signals with vectors representing respective ones of said complementary codes and transmitting said carrier signals to said receiver.

7. The method of claim 6 wherein said transformation matrix comprises a number rows corresponding to the number of said constellation symbols and a number of columns corresponding to the number of said complementary codes.

8. The method of claim 7 wherein said step of encoding includes the steps of encoding each of said data words into respective phase angles in accordance with gray scale encoding and associating the encoded phase angles with predetermined ones of said constellation symbols.

9. The method of claim 7 further comprising at said receiver the steps of

receiving a composite signal of the modulated carrier signals transmitted by said transmitter and regenerating individual ones of said vectors as a function of the received composite signal,

applying said kernel code to said regenerated vectors to generate vector elements representing individual ones of said complementary codes,

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forming, in accordance with a predetermined encoding matrix, pairs of said elements and associating individual ones of said pairs with respective ones of said constellation symbols, in which one element in each of said pair is taken as a complex conjugate so that said one element may be subtracted from the other element of the respective pair of elements,

deriving individual ones of said constellation symbols as a function of the subtracted result obtained from the associated ones of said pairs of elements, and

deriving each constellation symbol, φ_n, not associated with any one of said pairs as a function of the derived constellation symbols and a complementary code, θ_k, formed in part by that constellation symbol, φ_n.

10. The method of claim 9 wherein said constellation symbols represent respective phase angles, and wherein said step of deriving as a function of the subtracted result includes the steps of determining an angle for each of said subtracted result and associating the determined angle with the closes one of the phase angles and thus a respective one of the constellation symbols.

11. The method of claim 10 further comprising the step of identifying the data words respectively associated with the derived constellation symbols.

12. A transmitter for transmitting encoded data to a receiver, said transmitter comprising

means for encoding data words that are to be transmitted to said receiver into respective constellation symbols,

means for generating a plurality of complementary codes as a function of a selected kernel code and a predetermined transformation matrix of said constellation symbols, and

means for modulating a plurality of carrier signals with vectors representing respective ones of said complementary codes and transmitting said carrier signals to said receiver.

13. The receiver for receiving encoded data from a transmitter, said receiver comprising

means for receiving a composite signal of a plurality of signals transmitted by said transmitter and regenerating individual signal vectors as a function of the received composite signal,

means for applying a predetermined kernel code to said regenerated vectors to generate vector elements representing individual ones of said complementary codes,

means for forming, in accordance with a predetermined encoding matrix, pairs of said elements and associating individual ones of said pairs with respective ones of constellation symbols, in which one element in each of said pair is taken as a complex conjugate so that said one element may be subtracted from the other element of the respective pair of elements,

means for determining individual ones of said constellation symbols as a function of the subtracted result obtained from the associated ones of said pairs of elements, and

means for determining each constellation symbol, φ_n, not associated with any one of said pairs as a function of the derived constellation symbols and a complementary code, θ_k, formed in part by that constellation symbol, φ_n.
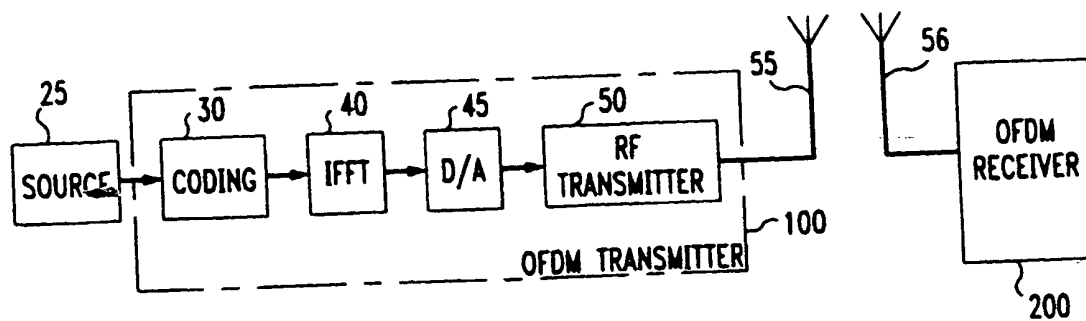
FIG. 1

FIG. 2